Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1328839610

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **October/November 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 7) and part **B** (Questions 8 to 12).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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Answer both parts **A** and **B**.

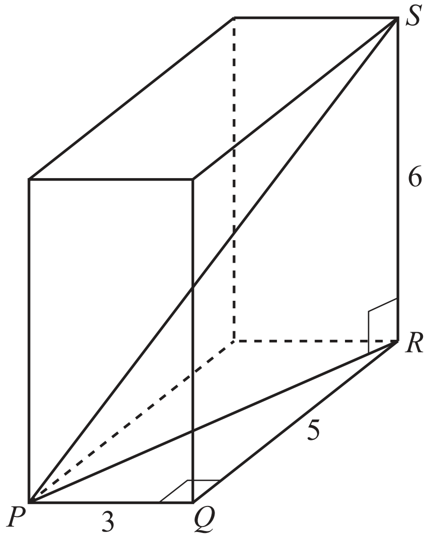
A INVESTIGATION (QUESTIONS 1 to 7)

PYTHAGOREAN SETS OF FOUR (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at finding the integer lengths of the sides of a cuboid that has an integer length for its diagonal.

1



NOT TO SCALE

The diagram shows a cuboid with sides of length 3, 5 and 6.

(a) Using Pythagoras' Theorem in triangle PQR gives $PR^2 = 3^2 + 5^2$.

Find the value of PR^2.

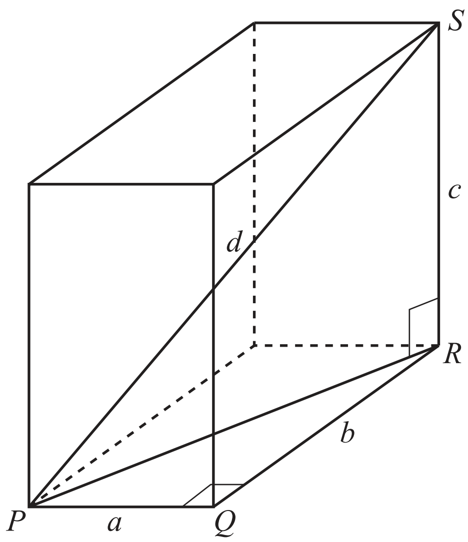
.... [1]

(b) Using Pythagoras' Theorem in triangle PRS gives $PS^2 = PR^2 + 6^2$.

Find the value of PS^2.

.... [2]

2



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The diagram shows a cuboid with sides of integer length a, b and c.
Its diagonal, PS, has integer length d.

(a) Use Pythagoras' Theorem in triangle PQR to write down an expression for PR^2 in terms of a and b.

.... [1]

(b) Use your answer to **part (a)**, and Pythagoras' Theorem in triangle PRS, to show that

$$d^2 = a^2 + b^2 + c^2.$$

[1]

3 A cuboid has sides of length a, b and c, where a, b and c are integers and $a \leqslant b \leqslant c$.
If the length of the diagonal, d, is also an integer then (a, b, c, d) is a *Pythagorean set of four*.

Use $d^2 = a^2 + b^2 + c^2$ to show that a cuboid with sides of length 4, 17 and 28 gives a Pythagorean set of four.
Complete the Pythagorean set of four.

(4, 17, 28, ...) [3]

4 **(a)** In a Pythagorean set of four (a, b, c, d) $d^2 = a^2 + b^2 + c^2$.

When $d = a + c$, show that $ac = \frac{b^2}{2}$.

[2]

(b) Explain why b **must** be even.

....

....

.... [2]

5 Here is a method for finding Pythagorean sets of four using **Question 4**:

- Choose any even integer for b.
- Calculate ac using **Question 4(a)**.
- Find all the possible pairs of integers for a and c, where $a < c$.

Use this method to find all the Pythagorean sets of four when you choose $b = 8$.

.... [7]

6 **(a)** Which one of these two sets is a Pythagorean set of four?

(18, 24, 72, 78) or (18, 24, 72, 84)?

Show how you decide.

(18, 24, 72, ...) [2]

(b) (ka, kb, kc, kd) is a Pythagorean set of four, where k is a positive integer greater than 1.

Use algebra to show that (a, b, c, d) must also be a Pythagorean set of four.

[2]

(c) (a, b, c, d) is a *basic* Pythagorean set of four if the numbers a, b, c and d have no common factor greater than 1.

Find the basic Pythagorean set of four for your answer to **part (a)**.

.... [2]

7 The method in **Question 5** to find Pythagorean sets of four is:

- Choose any even integer for b.
- Calculate ac using **Question 4(a)**.
- Find all the possible pairs of integers for a and c, where $a < c$.

Use this method to find two basic Pythagorean sets of four where the smallest integer, a, is 2.

(........... , ... , ... , ...) and (........... , ... , ... , ...) [5]

B MODELLING (QUESTIONS 8 to 12)

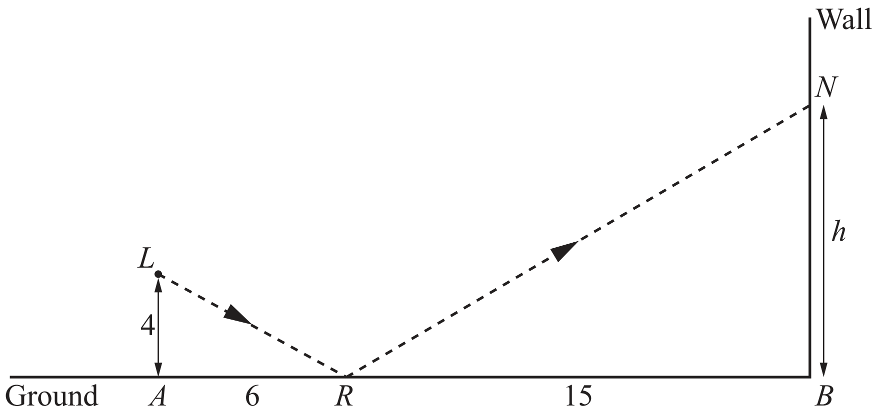
REFLECTING A LASER BEAM (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at models for the height of the image of a reflected laser beam on a vertical wall.
In this task all the measurements are in metres.

The diagram shows, by a dashed line, the side view of the path of a laser beam.

The laser beam
- starts at source L
- travels to a point R on horizontal ground AB
- reflects at the point R so that angle LRA = angle NRB
- travels to N, its image, on a vertical wall.



NOT TO SCALE

The height of L above the horizontal ground is $LA = 4$.
The height of N above the horizontal ground is $NB = h$.
$AR = 6$ and $RB = 15$.

8 Complete the statements to show that $h = 10$.

Triangle LRA is similar to triangle NRB.

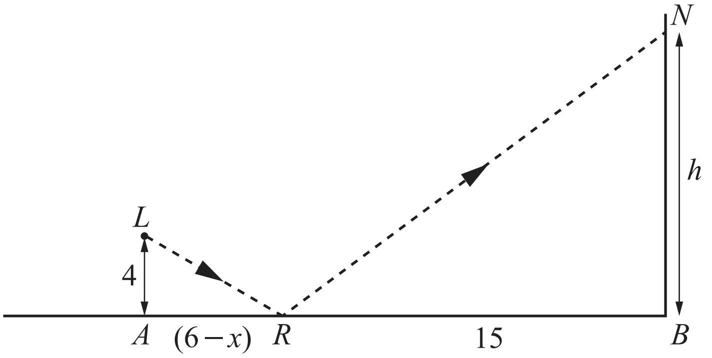
$$\frac{h}{4} = \frac{15}{\dots\dots}$$

$$h = \frac{\dots\dots \times \dots\dots}{\dots\dots} = 10$$

[2]

9 The laser source, L, can move towards or away from the wall.
It now moves x metres to the right so that AR becomes $6-x$ metres.
The point R does not move.
The other given measurements remain the same.

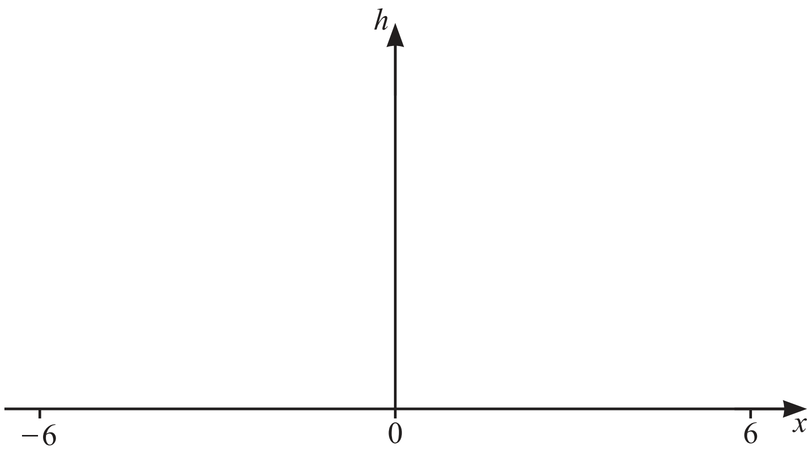
When the laser beam reflects at R, triangle LRA and triangle NRB will always be similar.



(a) Use the method in **Question 8** to find a model for h in terms of x.

.... [2]

(b) Sketch the graph of h against x for $-6 < x < 6$.



[2]

(c) **(i)** Write down the equation of the vertical asymptote.

.... [2]

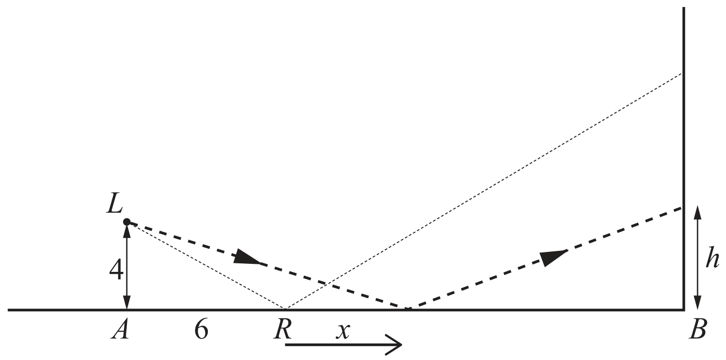
(ii) Give a reason why there is a vertical asymptote.
Refer to the path of the laser beam.

....

.... [1]

10 The laser source, L, now stays fixed.
At the start $AR = 6$ and $RB = 15$.
The point R then moves x metres towards B along the ground.
The dashed line shows the path of the laser beam.

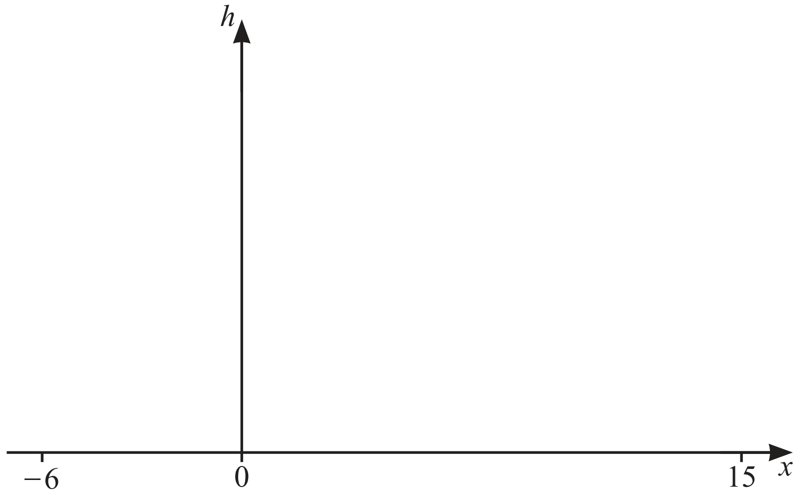
NOT TO SCALE



(a) Show that $h = \dfrac{60-4x}{6+x}$.

[2]

(b) Sketch the graph of $h = \dfrac{60-4x}{6+x}$ for $-6 < x \leqslant 15$.



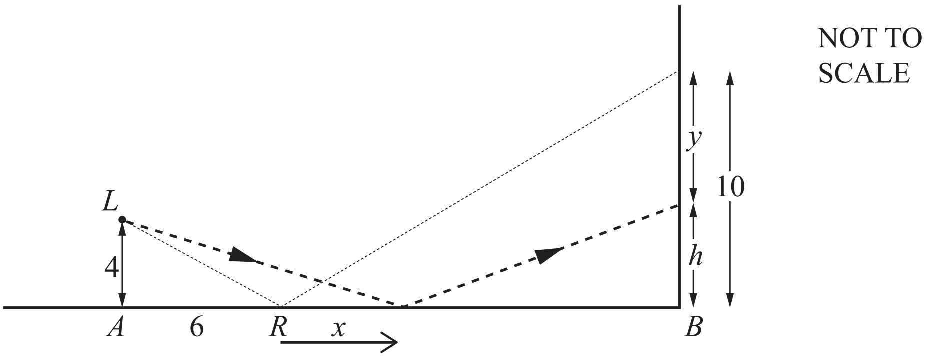
[3]

(c) When the point R has moved x metres towards B, the height, h, is 6.

Find the value of x.

.... [2]

11



At the start, when $AR = 6$, the height of the image is 10.
After the point R moves x metres, the height of the image is $h = \frac{60-4x}{6+x}$.
y is the change in the height of the image, so $y = 10 - h$.

(a) Show that a model for y is $y = \frac{14x}{6+x}$.

[3]

(b) (i) When the point R moves one metre to the left, away from B, $x = -1$.

Use the model in **part (a)** to find the change in height of the image.

.... [2]

(ii) The point R moves an **additional** one metre to the left, away from B.

(a) Write down the value of x.

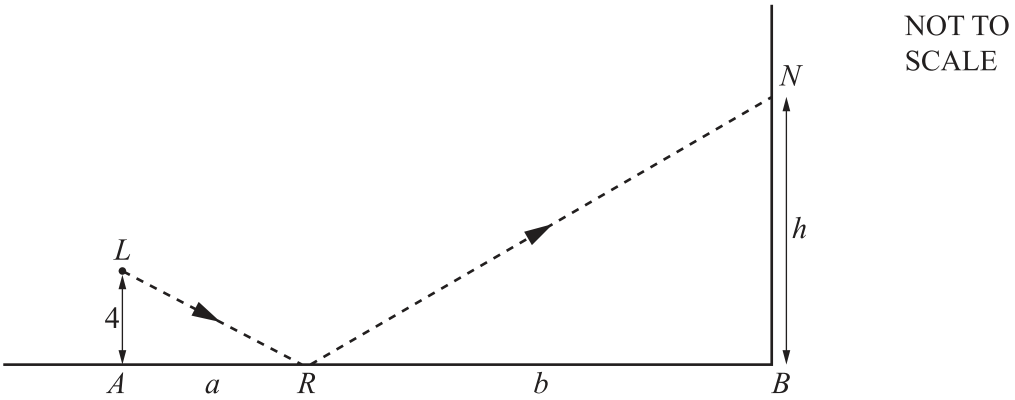
.... [1]

(b) Find the **additional** change in height.

.... [3]

Question 12 is printed on the next page.

12



(a) Find h in terms of a and b.

.... [2]

(b) The point R moves x metres to the right, towards B, along the horizontal ground.
y is the change in h.

Find a model for y in terms of a, b and x.
Do not simplify your answer.

.... [3]